PARTICLE VELOCITY IN A FLUIDIZED BED

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The particle velocity in a fluidized bed of granular material is determined from an equation obtained by analyzing the forces applied to each particle of the bed. The coefficient characterizing the degree of confinement of the particles in the bed is determined experimentally.

A knowledge of the particle velocity in a fluidized bed of granular material makes possible a better understanding of the heterogeneous processes that take place in such a bed.

The velocity of the solids is needed to calculate a number of hydraulic and diffusional parameters and in order to take into account mixing of the phases (solid and gas) in the bed and its effect (mixing) on the motive force of the process.

Unfortunately, none of the published papers include formulas that would permit an accurate calculation of the particle velocity as a function of the specific hydrodynamic situation [4–8, 10].

At present, there are essentially two ways of determining the particle velocity in a fluidized bed: high-speed motion-picture photography and the isotope-tagged particle method [4, 7, and 10].

The latter method, although undoubtedly promising, is still inefficient not only owing to the complexity of the equipment and the safety measures required but also owing to the difficulties in reconstructing the three-dimensional motion of a tagged particle from the signals received from some system of pulse transducers.

Accordingly, we decided to employ the method of high-speed motion-picture photography* in conjunction with a vessel with transparent walls.

The particle velocity u in the fluidized bed was treated primarily as a function of the velocity of the fluidizing agent and the geometrical dimensions of the bed, i.e.,

$$u = f(w_*, w_0, H, D_a, \ldots). \tag{1}$$

The experiments were performed using as the solid phase two sand fractions with a mean (reduced) particle diameter of 250 and 500 μ ($\rho_g = 2500$ kg/m^3) and two fractions of silica gel (grade KSM) with a mean grain diameter of 3.5 and 4 mm ($\rho_g = 2250$ kg/m^3). The working range of air velocities was from 0.1 to 0.8 m/sec for the sand and from 1.5 to 5.0 m/sec for the silica gel.

The experiments were carried out in glass vessels with inside diameters of 60 and 300 mm. The height of the vessel could be increased by assembling several sections each 500 mm long.

The particle velocity u was determined by examining the film frame-by-frame in an optical instrument of the "Mikrofot" type (at a tenfold magnification of the image) from the formula

$$u = \Delta l/\tau = \Delta l \nu/n. \tag{2}$$

It is clear from the photographs in Fig. 1 how we determined the displacements of the particles from frame to frame.

For silica gel the particle velocity was determined both from the upper boundary of the bed (in this case the boundary of the bed is considered as the locus of individual particles moving in the bed at a certain velocity u) and directly from the displacements of the individual particles in the bed. For the fine fractions (sand) the particle velocity was mainly determined from the upper boundary of the bed or a cavity in the bed. Determination of the particle velocities from the upper boundary of the center of the bed makes it possible to find these velocities not only at the walls of the apparatus but over the entire cross section of the bed.

Determination of the particle velocity from Eq. (2) gives a broad spectrum of values of u. This indicates that the individual particle velocities in a granular gas-fluidized bed may be regarded as random quantities lying within the interval [5, 6] $u_{min} < u < u_{max}$.

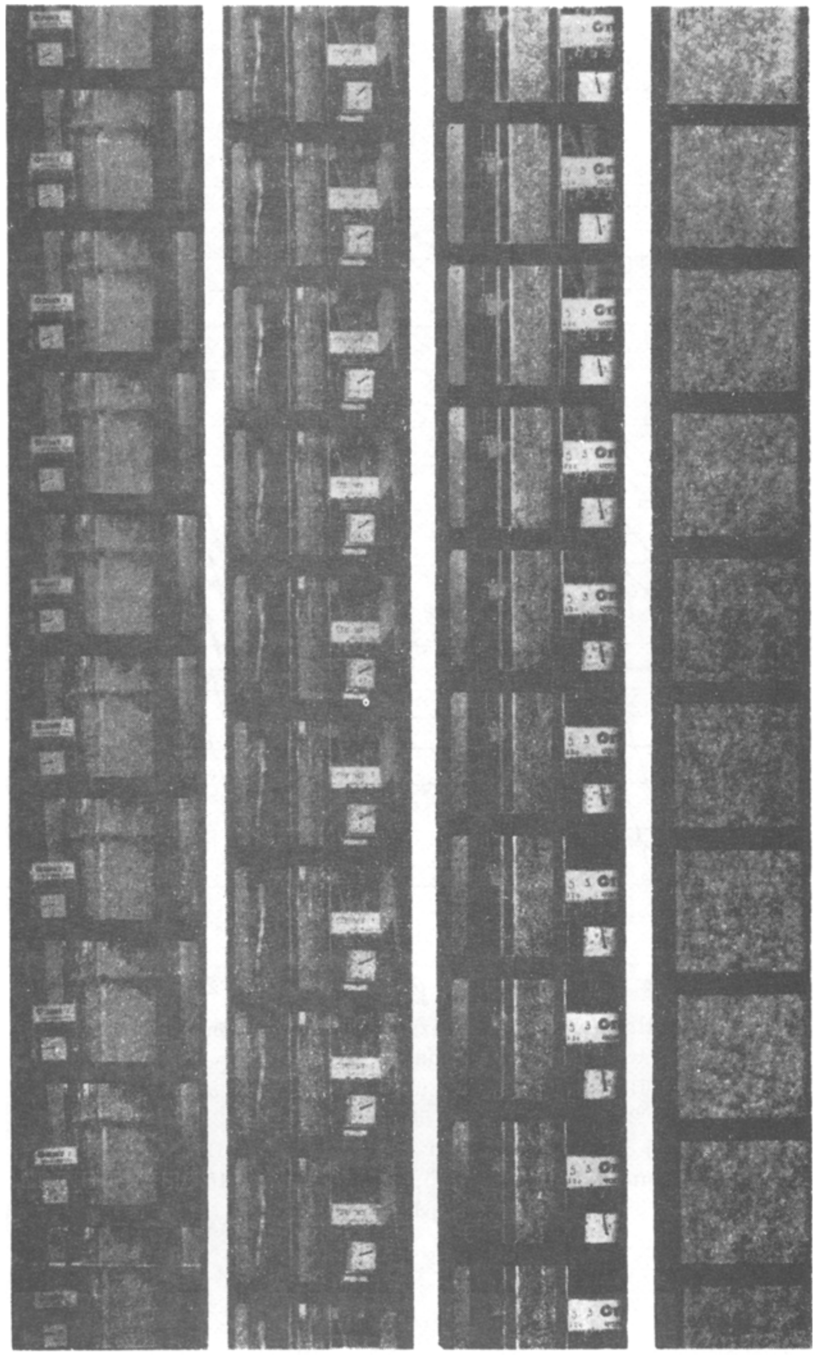
The particle velocities can be described sufficiently accurately by a normal distribution law. The same velocity distribution is also observed in connection with settling in a suspended bed [6]. Accordingly, the particle velocity for each hydraulic regime was determined as the arithmetic mean of the results of analyzing 500–750 frames, including the duplicate experiments and close-up photographs shown in Fig. 1.

Figures 2 and 3 present the results of an experimental determination of the particle velocity as a function of the working velocity of the fluidizing agent and the height of the fluidized bed.

Clearly, the vertical particle velocity increases with increase in the gas flow rate, which indicates a proportional dependence on the gas velocity in the spaces between solids; moreover, under the same hydrodynamic conditions (equality of H_0/D_a and fluidizing agent velocities) the particle velocity is greater in the large-diameter than in the small-diameter apparatus.

The data of Fig. 3 confirm the existence of an inverse relationship between particle velocity and the height of the fluidized bed.

*The films were obtained with the assistance of I. Ya. Tikhomirov of the Moscow Order of Lenin Motion-Picture Studio of Popular Science Films.



a b c d

Fig. 1. Photographs of fluidized bed in vessels 300 mm (a) and 60 mm (b, c, d) in diameter.

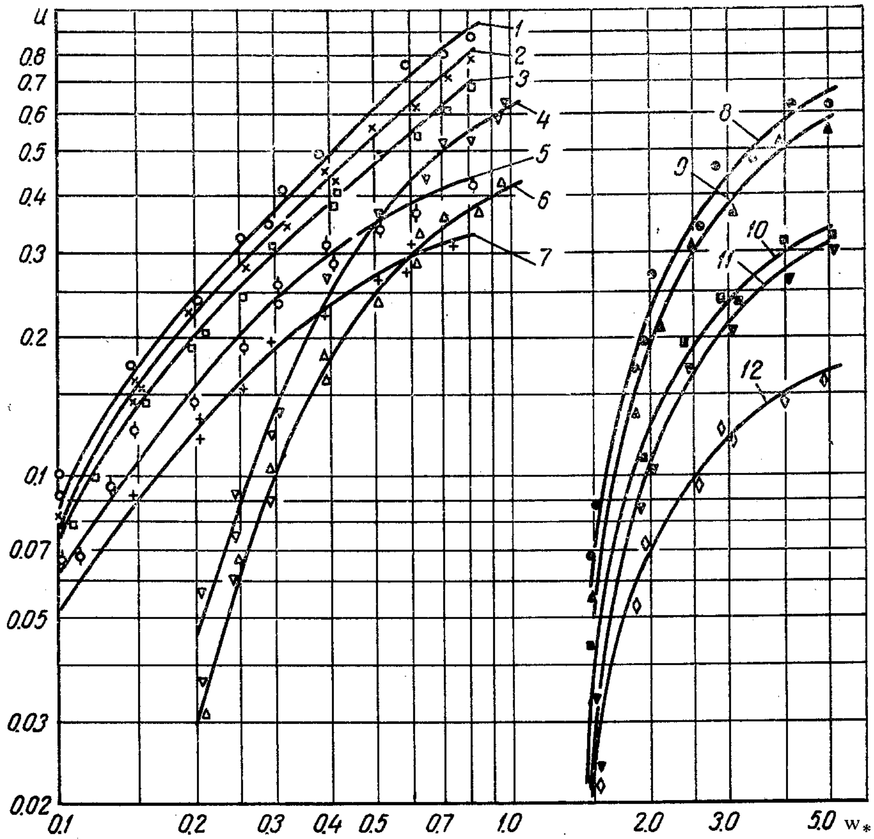


Fig. 2. Particle velocity u, m/sec, as a function of the working gas velocity w_*, m/sec: for quartz sand: 1) D_a = 300 mm, d_g = = 250 μ and H_0/D_a = 1; 2) D_a = 60 mm, d_g = 250 μ and H_0/D_a = = 0.5; 3) 60, 250 and 1; 4) 60, 500 and 1; 5) 60, 250 and 3; 6) 60, 500 and 2; 7) 60, 250 and 5; for KSM silica gel: D_a = 60 mm; 8) d_g = 3.5 mm and H_0/D_a = 0.5; 9) 4.0 and 0.5; 10) 3.5 and 1; 11) 4.0 and 1; 12) 3.5 and 2.

Moreover, the experimental data reveal that the particle velocity is inversely proportional to the diameter of the grain and its specific weight, which is in good agreement with the data of other authors [4–8, and 10].

In deriving the formula for the particle velocity the motion of the particle was regarded as a result of the action of the gravity force G = mg and the resistance of the medium

$$R = \xi F_p \frac{\rho}{2} (w_f - u)^2.$$

The equilibrium conditions of the forces producing motion of the particles in a fluidized bed with a vertical gas flow penetrating the bed at velocity w_f, assuming that the particles are spherical, can be written in the form of the following differential equation (d'Alembert equation [5, 6, 8]):

$$m \frac{du}{d\tau} = -mg + \frac{1}{2} \xi F_p \rho (w_f - u)^2 = -mg + \lambda (w_f - u)^2, \tag{3}$$

where

$$\lambda = \xi F_p \frac{\rho}{2} = \xi \rho \frac{\pi d_g^2}{8}.$$

The drag should be represented as the sum of two forces. Then Eq. (3) can be written in the following form:

$$m \frac{du}{d\tau} = -mg + (\varphi\lambda + \psi\lambda)(w_f - u)^2, \tag{4}$$

where ψ is the fraction of the drag expended by the flow of fluidizing agent in communicating to the particle an acceleration $du/d\tau$, and φ is the fraction of the drag expended by the flow of fluidizing agent in counteracting the gravity force

$$G = mg = \varphi\lambda (w_f - u)^2,$$
$$\varphi + \psi = 1.$$

This means that in a fluidized bed of solid granular material the particles move under the action of the dynamic head of a stream of fluidizing agent, part of which is expended in counteracting gravity:

$$m \frac{du}{d\tau} = \psi\lambda (w_f - u)^2. \tag{5}$$

It is necessary to consider several special cases.

1. $\psi = 0$, $\varphi = 1$, i.e., $m du/d\tau = 0$ is the case when under the action of gravity the particle acquires a steady critical velocity v_r under confined conditions (case of suspension of a particle in the space between grains filled with fluidizing medium); in this case $w_f = v_r$ and $u = 0$. Then Eq. (3) takes the following particular form:

$$mg = \lambda v_r^2, \tag{6}$$

from which follows the important result

$$m/\lambda = v_r^2/g \approx 0.1 v_r^2. \tag{6a}$$

If $w_f \neq v_r$, then the particle velocity has the following particular value:

$$u = w_f - v_r. \tag{7}$$

2. $\psi = \varphi = 1/2$. In this case the particle can move with a velocity

$$u \approx w_f - 1.41 v_r. \tag{7a}$$

3. $\psi = 1$, $\varphi = 0$—this case does not occur in practice.

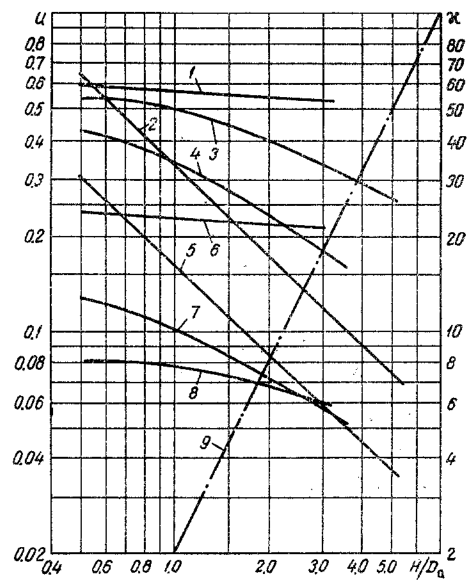


Fig. 3. Particle velocity u, m/sec, as a function of the height of the fluidized bed: 1 and 6) quartz sand: D_a = 300 mm, d_g = = 250 μ, w_* = 0.5 and 0.2 m/sec; 2 and 5) silica gel: D_a = 60 mm, d_g = 4.0 mm, w_* = = 5.0 and 1.5 m/sec; 3 and 8) quartz sand: 60 mm, 250 μ, 0.5 and 0.1 m/sec; 4 and 7) quartz sand: 60 mm, 500 μ, 0.4 and 0.2 m/sec; 9) coefficient ϰ as a function of the ratio H/D_a in formula (10a).

In the general case $\psi \neq 0$ and $\varphi \neq 0$. Then the solution of differential equation (5) with Eq. (6a) is found in the form

$$\frac{1}{w_f - u} = \psi \frac{g}{v_r^2} \tau + C = \frac{g\tau}{\varkappa v_r^2} + C. \tag{8}$$

The constant of integration C is found from the initial conditions: u = 0 at τ = 0. Hence

$$C = 1/w_f. \tag{8a}$$

Then the vertical particle velocity is given by

$$u = g w_f^2 \tau / (\varkappa v_r^2 + g w_f \tau), \tag{9}$$

or making the substitution $\tau = H/w_f$, by

$$u = g w_f H / (\varkappa v_r^2 + gH), \tag{9a}$$

where $\varkappa = 1/\psi$ is a dimensionless coefficient determining the complex dependence of the critical velocity under confined conditions with allowance for the critical velocity distribution over the height of the fluidized bed.

Coefficient $\varkappa$ as a Function of the Characteristic Parameters of the Bed

Quartz sand, d_g = 250 μ										
w, m/sec	0.225	0.305	0.378	0.445	0.508	0.630	0.745	0.857	0.960	1.05
u, m/sec	0.082	0.156	0.218	0.282	0.336	0.448	0.536	0.630	0.712	0.775
H, m	0.032	0.035	0.038	0.041	0.044	0.049	0.055	0.060	0.067	0.075
H/D_a	0.54	0.59	0.064	0.68	0.73	0.81	0.91	1.00	1.11	1.25
$\varkappa$	0.59	0.70	0.84	0.94	1.06	1.30	1.64	2.02	2.45	3.15
u, m/sec	0.076	0.145	0.202	0.258	0.307	0.405	0.495	0.560	0.630	0.675
H, m	0.065	0.071	0.076	0.082	0.087	0.097	0.11	0.12	0.135	0.15
H/D_a	1.08	1.18	1.28	1.36	1.46	1.62	1.82	2.03	2.22	2.50
$\varkappa$	2.30	2.81	3.30	3.37	4.15	5.15	6.66	8.00	9.60	12.5
u, m/sec	0.060	0.112	0.155	0.195	0.226	0.295	0.344	0.390	0.423	0.440
H, m	0.195	0.212	0.232	0.245	0.265	0.290	0.330	0.362	0.400	0.450
H/D_a	3.24	3.55	3.85	4.09	4.35	4.85	5.47	6.05	6.70	7.50
$\varkappa$	21.0	24.5	29.0	33.3	37.5	47.8	62.0	71.0	89.2	113.0
Quartz sand, d_g = 500 μ										
w, m/sec	0.506	0.600	0.670	0.856	1.00	1.16	1.30	1.42	1.66	
u, m/sec	0.045	0.100	0.140	0.250	0.330	0.410	0.470	0.530	0.635	
H, m	0.061	0.062	0.065	0.068	0.072	0.074	0.078	0.082	0.090	
H/D_a	1.01	1.03	1.08	1.12	1.20	1.24	1.30	1.37	1.50	
$\varkappa$	2.02	2.15	2.35	2.65	2.85	3.10	3.47	3.77	4.55	
Silica gel, d_g = 4.0 mm										
w, m/sec	3.75	4.20	4.47	5.12	5.70	6.77	7.80			
u, m/sec	0.019	0.074	0.105	0.160	0.215	0.285	0.340			
H, m	0.061	0.063	0.065	0.070	0.076	0.088	0.100			
H/D_a	1.01	1.05	1.08	1.17	1.26	1.46	1.67			
$\varkappa$	1.98	2.17	2.20	2.80	3.15	4.25	5.55			

Values of $\varkappa$ were determined from expression (9) after transformation and the substitution of $\tau = H/w_f$:

$$\varkappa = \frac{gH}{uv_r^2}(w_f - u) \approx 900\frac{H}{v^2}(w_f/u - 1). \qquad (10)$$

As may be seen from Fig. 4. and the table, the complex dependence of the coefficient $\varkappa$ on the characteristic parameters of the bed can be expressed in the first approximation as

$$\varkappa = 2H^2/D_a^2. \qquad (10a)$$

Equation (9) shows that the particle velocity in the bed is proportional to the actual velocity of the fluidizing agent and the time of action of the flow energy on the moving particle and roughly inversely proportional to the square of the critical velocity under confined conditions when the degree of confinement is represented by the coefficient $\varkappa$.

The model adopted and expression (9) are valid for $\tau \le H/w_f$, i.e., for the time during which the volume element of gas in which the transported particle is located traverses a path equal to the height of the bed. The path of the particle in time τ may be determined if expression (9) is integrated again with respect to the variable τ for $S = 0$ at $\tau = 0$:

$$S = w_f\,\tau - \frac{1}{g}\varkappa v_r^2 \ln\left(1 + \frac{g w_f \tau}{\varkappa v_r^2}\right) \quad \text{at} \quad \tau \leqslant H/w_f. \qquad (11)$$

The above reasoning is valid for the case of a particle falling (settling) in a fluidized bed (counterflow of solid and gas phases) if the quantities are taken with the opposite sign. Thus, in order to calculate the particle velocity in a homogeneous fluidized bed it is necessary to use Eq. (9) and determine the path of the particle from Eq. (11).

The actual mean velocity of the fluidizing agent in the spaces between the grains is equal to the working velocity divided by the mean voidage:

$$w = \frac{w_*}{\varepsilon} \approx \frac{V_{hr}\,H}{900\pi D_a^2[H - (1-\varepsilon_0)H_0]}. \qquad (12)$$

The critical velocity v is determined from the conditions when the lift force acting on the individual particle becomes equal to the weight of that particle [1–9]:

$$v^2 = \frac{4}{3}\frac{gd_g}{\xi}\frac{\rho_g - \rho}{\rho}. \qquad (13)$$

Since in order to calculate the critical velocity under confined conditions it is necessary to know the drag coefficient, it is more convenient to use the relation [3]

$$v_r^2 = \alpha v^2 = \frac{3}{4}\frac{v^2\varepsilon_0^3\xi_f}{1.75} \approx 0.011\,v^2, \qquad (14)$$

where $\varepsilon_0 \approx 0.4$; $\xi_f \approx 0.4$.

In general form [1–9]

$$\xi\,\mathrm{Re}^2 = \frac{4}{3}\mathrm{Ar}.$$

At a fluid velocity less than w_0/ε_0, conditions correspond to the percolation regime; therefore as the

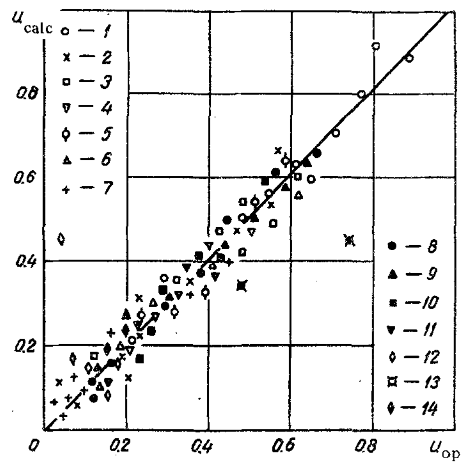


Fig. 4. Correlation graph: 1,2,3,4,5, 6,7) for quartz sand; 8,9,10,11) silica gel; 12) data of [4]; 13) [7]; 14) [10].

reduced actual velocity of the fluidizing agent in the spaces between the grains it is necessary to use the quantity

$$w_f = w - \frac{w_0}{\varepsilon_0} = \frac{w_p}{\varepsilon} - \frac{w_0}{\varepsilon_0}. \qquad (15)$$

NOTATION

u is the velocity of solid particles along the walls of the vessel at times τ; m is the particle mass; Δl is the displacement (path) of the particle in time τ; ν is the film speed (frame frequency); n is the number of frames in which the observed particle traverses a path equal to Δl; w_* is the velocity of fluidizing agent referred to total cross section of the vessel; w_0 is the fluid velocity at the onset of fluidization; w is the actual velocity of the fluidizing agent; ε is the bed voidage; v is the critical particle velocity in the unconfined space; v_r is the critical particle velocity under confined conditions; ξ is the particle drag; F_p is the drag surface (maximum cross section) of the particle; ρ is the density of the medium (fluidizing agent); ρ_g is the density of solid-phase particles; d_g is the grain diameter; g is the acceleration of gravity; $\varkappa$ is the coefficient expressing degree of confinement of particles in bed; H is the height of the bed of granular material at the onset of fluidization; D_a is the diameter of the apparatus; V_{hr} is the rate of flow of the fluidizing agent through bed per hour; w_f is the actual reduced velocity of the fluidizing agent in the spaces between the grains.

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